Chapter 1 Review of Some Basic Physics Concepts

A The Electromagnetic Spectrum

An understanding of the electromagnetic spectrum, and electromagnetic radiation is the first essential item we must address. Propagating electromagnetic energy may either be considered to be a special configuration of electric $(\vec{E})$ and magnetic $(\vec{B})$ fields traveling outward from a source or it may be described as a stream of particle-like objects called photons. In either case the radiation originates from electric charges undergoing accelerations or sudden transitions in their energy levels. The description of electromagnetic radiation (and its interaction with matter) as a wave phenomenon depends on the concepts of wavelength and frequency. These two quantities are related by

$$\lambda f = c \qquad \text{(Eqn. 1.1)}$$

Where λ = wavelength (in meters)
f = Frequency in Hz (cycles/sec)
c = Phase velocity of the wave (in m/sec)

For vacuum the value of c = 2.998 x 10^8 (m/sec), an important constant of physics. The range of wavelengths which will be of interest to us covers a very large range of about 20 orders of magnitude as can be seen in Figure 1.1. (Initial study question: Where does x-band radar fall in this diagram?)

The connection between these two descriptions (wave-like and particle-like) for electromagnetic radiation generally merge in the concept of the photon, which is then often described as a particle with wave-like properties. The energy of an individual photon is given by

$$E = hf \text{ (Joules or eV)} \qquad \text{(Eqn. 1.2)}$$

where f = frequency of the EM wave (in Hz) and

$$h = \text{Planck's Constant} = \begin{Bmatrix} 6.626 \times 10^{-34} \text{ Joule-seconds} \\ 4.136 \times 10^{-15} \text{ eV-seconds} \end{Bmatrix}$$

The photon energy, E, is determined by the frequency of the electromagnetic radiation. The higher the frequency, the higher the energy. Photons move at the speed of light, as expected for electromagnetic radiation. They have zero rest mass, however, so the rules of special relativity are not violated.

The electron-volt (eV) is a convenient unit of energy and is related to the usual unit (Joule) by:

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ Joules} \qquad \text{(Eqn. 1.3)}$$

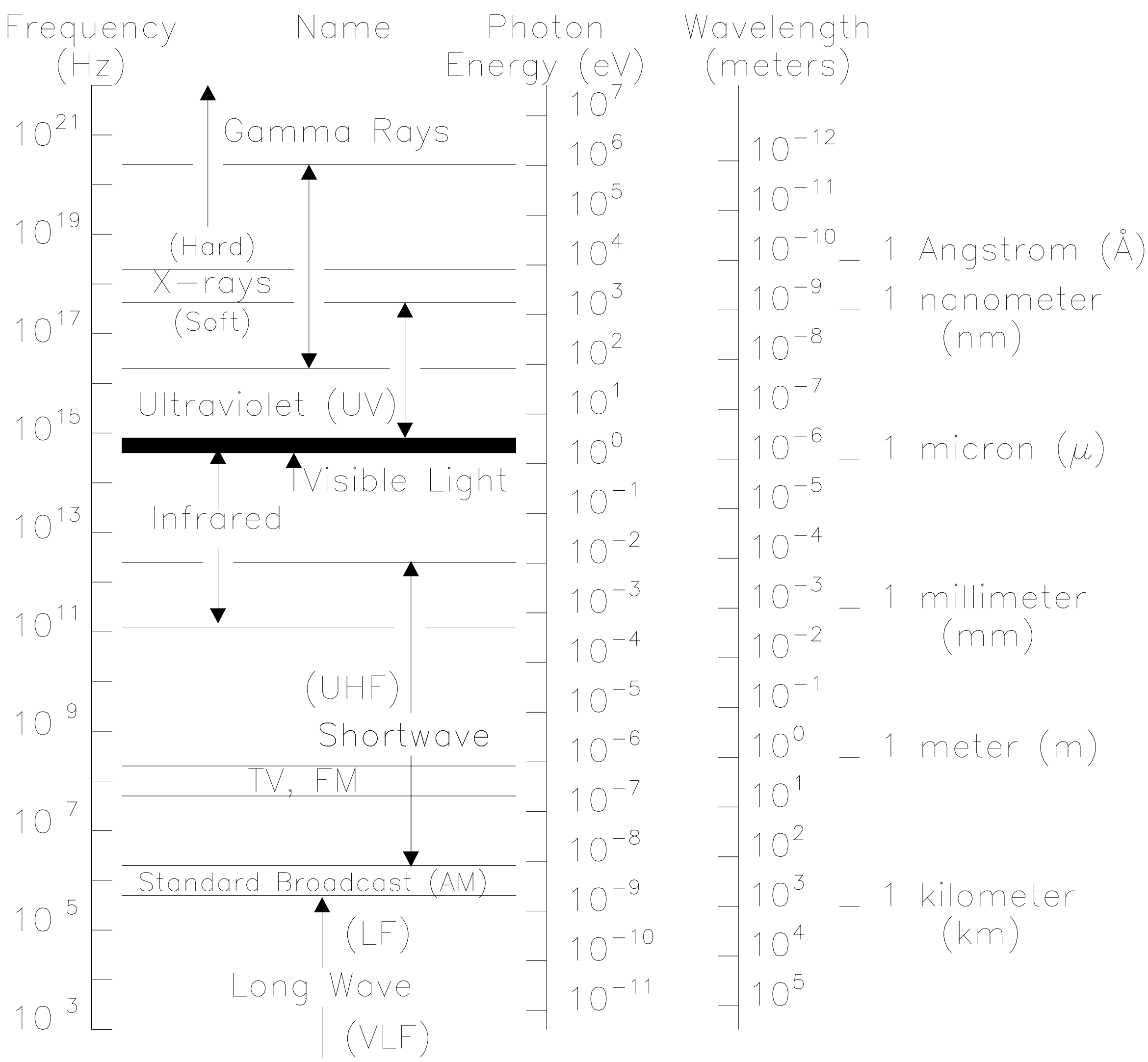

Figure 1.1 The spectrum of electromagnetic radiation

The question is often asked: What is electromagnetic radiation really? Is it a wave or is it a stream of particles? To answer this question we must perform some carefully designed experiment which will tell us. Depending on the experiment we find that we can get either answer, but only one at a time. It will also be found that in general the wave aspects dominate at frequencies below about 10^{15} Hz and the particle aspects at higher frequencies. In the visible part of the spectrum both descriptions are useful.

Example:

The energy of the photons making up an electromagnetic wave (light wave) in the visible part of the spectrum (green color) is

$$E = hf = (4.14 \text{ x } 10^{-15})\ (6 \text{ x } 10^{+14}) = 2.48 \text{ eV}$$

which is on the order of (or slightly less than) typical atomic binding energies.

Energies of typical x-ray photons are in the 10^4 to 10^5 eV range, while the photons of a 100 MHz radio signal are only about 4×10^{-7} eV.

B Sources of Electromagnetic Radiation

There are 3 major sources of electromagnetic radiation which are of interest to us in this course:
(a) Individual atoms or molecules which radiate line spectra
(b) Hot, dense bodies which radiate a continuous "black-body" spectrum
(c) Energetic electrons which radiate x-rays when accelerated (or decelerated)

Each share a common element, in the end - that electromagnetic radiation is ultimately produced by electrons which are being accelerated (classically), or changing energy (quantum mechanics). The former case is most obvious in case c, the latter in case a. These sources of radiation are now considered in somewhat more detail.

1 Line spectra

Line spectra are emitted by single atoms or molecules. An atom or molecule which is reasonably isolated (such as in a gas at ordinary temperatures and pressures) will **radiate** a discrete set of frequencies called a line spectrum, as shown in Figure 1.2. If, on the other hand, we pass radiation having a continuous spectrum of frequencies through a gas we find that a discrete set of frequencies is **absorbed** by the gas leading to a spectrum of discrete absorption lines

The wavelengths radiated (absorbed) are characteristic of the particular atom or molecule and thus represent a powerful tool for determining the composition of radiating (or absorbing) gases. Much of our knowledge of the chemical composition of stars (including our sun) comes from detailed analysis of such line spectra.

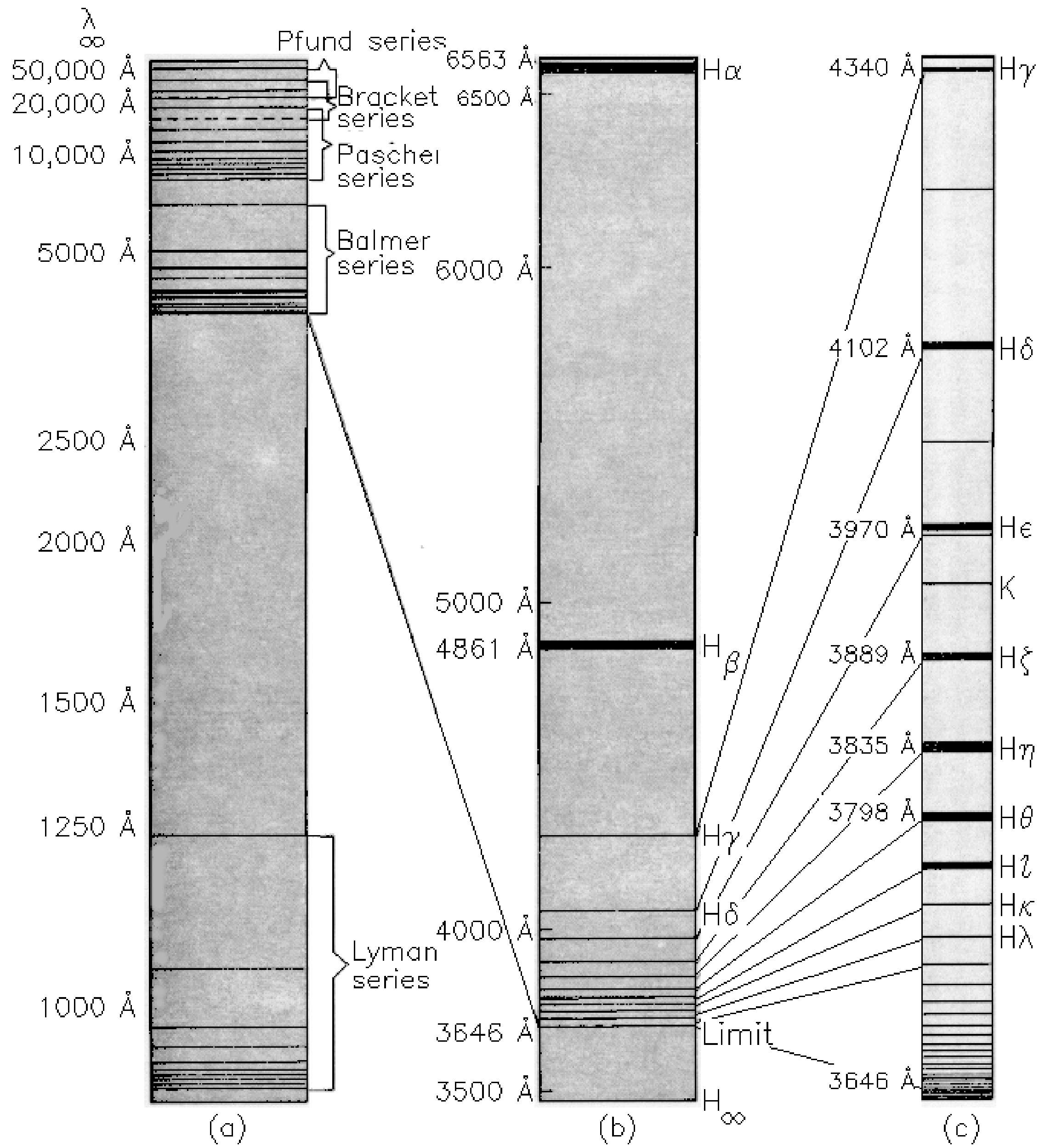

Figure 1.2 (a) The spectrum of atomic hydrogen consists of the Lyman series in the ultraviolet, the Balmer series in the visible, and several series in the infrared region. (b) The Balmer series in greater detail. (c) A portion of the spectrum of the star ζ Tauri showing more than 20 lines of the Balmer series.

Original citation: A. W. Smith and J. N. Cooper, "Elements of Physics", 7th edition, copyright 1964, McGraw Hill Book Co.

Found in: Introduction to Modern Physics, F. K. Richtmyer, E. H. Kennard, and John N. Cooper, page 229, 6th edition, 1969.

Derivation of the Bohr Atom

The existence of line spectra can be explained by means of the first 'quantum' model of the atom, developed by Bohr in 1913. Although the Bohr model of the hydrogen atom was eventually replaced, it yields the correct values for the observed spectral lines, and gives a substantial insight into the structure of atoms in general. The following derivation has the objective of obtaining the energy levels of the Bohr atom. If we can obtain the energy levels, we can reproduce the hydrogen atom spectra.

The derivation proceeds with three major elements: first, use force balance to relate the velocity to the radius of the electron orbit, then use a quantum assumption to get the radius, then solve for the energies.

Assumption 1: The atom is held together by the Coulomb Force

It is an experimental fact that the force F between two point charges q_1 and q_2 separated by a distance r is given by:

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} \qquad \text{(Eqn. 1.4a)}$$

where $\frac{1}{4\pi\varepsilon_o} = 8.99\times10^9 \left(\frac{\mathrm{N\,m}^2}{\mathrm{C}^2}\right)$ and q_1 and q_2 are in units of Coulombs. The distance, r, is in meters of course. Note that the charges may be positive or negative.

For a single electron atom, we take the charge of the nucleus, q_1, to be +Ze, where Z is the atomic number of the atom (the number of protons in the nucleus). Z equals 1 for hydrogen. The charge of the electron, q_2, is -e. Substituting the values into (1.4) above we obtain:

$$F = -\frac{Ze^2}{4\pi\varepsilon_0 r^2} \qquad \text{(Eqn. 1.4b)}$$

The minus sign on the force term means the force is 'inward', or attractive.

Assumption 2: The electron moves in an elliptical orbit around the nucleus (as in planetary motion).

Let us assume that the electron moves in a circular orbit around the nucleus.

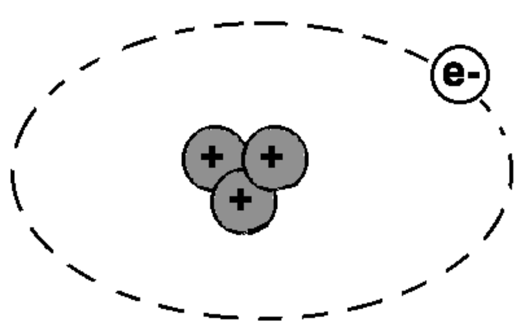

Figure 1.3 - Bohr atom model, Z=3

Then Newton's second Law (F=ma) (here: setting the Coulomb force equal to the centripetal force) can be written as:

$$\frac{-Ze^2}{4\pi\varepsilon_0 r^2} = -mv^2 / r \qquad \text{(Eqn. 1.5)}$$

and we can now solve for the radius vs velocity.

Assumption 3: Quantized angular momentum

Bohr now introduced the first of his two new postulates, namely that the only allowed orbits were those for which the angular momentum, L, was given by:

$$L = mvr = n\hbar \qquad \text{(Eqn. 1.6)}$$

where:

m = electron mass; v = velocity; r = radius of the orbit; n = an integer (1,2,3,..); and

$$\hbar = \frac{h}{2\pi} = 1.054 \times 10^{-34}\ (\text{Joule-seconds}) = 0.658 \times 10^{-15}\ (\text{eV-seconds})$$

where “h” is simply Planck's constant, as before.

(One suggestion for a physical basis for this assumption is that if you view the electron as a wave, with wavelength $\lambda = h/\mathrm{p} = h/\mathrm{mv}$, then an integral number of wavelengths have to fit around the circumference defined by the orbit, or $n \bullet h/\mathrm{mv} = 2\pi r$. Otherwise, the electron "interferes" with itself. This all follows as a corollary to the idea that an electromagnetic wave is a particle with energy $\mathrm{E}=hf$ as above, and hence the momentum of a photon is $\mathrm{p}=\mathrm{E}/\mathrm{c} = hf/\mathrm{c} = h/\lambda$.)

This is sufficient to give us:

$$v_n = \frac{n\hbar}{m r_n} \qquad \text{(Eqn. 1.7)}$$

for the velocity of the electron in its orbit. Note that there is an index n, for the different allowed orbits. It follows that:

$$\frac{m\, v_n^2}{r_n} = \frac{Ze^2}{4\pi\varepsilon_0\, r_n^2} = \frac{mn^2\,\hbar^2}{m^2\, r_n^3} \qquad \text{(Eqn. 1.8)}$$

Upon solving for the radius of the orbit (r_n), we get:

$$r_n = \frac{n^2\,\hbar^2}{m} \times \frac{4\pi\varepsilon_o}{Ze^2} = n^2\left(\frac{4\pi\varepsilon_o\,\hbar^2}{Zme^2}\right) \qquad \text{(Eqn. 1.9)}$$

$$r_n(\text{meters}) = n^2 \times 0.528 \times 10^{-10} / Z .$$

This only works for one electron atoms (H and He^+ as a practical matter), but within that restriction, it works fairly well. For hydrogen (Z=1) we get the Bohr radius, $r_1 = 0.528 \times 10^{-10}$ meters as the radius of the smallest orbit. The radius of the Bohr hydrogen atom is half an Angstrom. What is the radius of the orbit for the sole electron in He+ (singly ionized helium, Z=2)?

Now we can solve for the energy levels.

The potential energy associated with the Coulomb force is:

$$U = \frac{q_1 q_2}{4\pi \varepsilon_o r} \qquad \text{(Eqn. 1.10a)}$$

taking U (r = ∞) = 0, and plugging in for the charges, we get

$$U = -\frac{Ze^2}{4\pi\varepsilon_0 r} \qquad \text{(Eqn. 1.10b)}$$

A negative potential energy means the electron is in a potential 'well'. Given this expression for the potential energy, we need a similar expression for kinetic energy.

The kinetic energy, T, is easily obtained from equation 1.5:

$$T = \tfrac{1}{2} mv^2 = \frac{1}{2}\frac{Ze^2}{4\pi \varepsilon_0 r} \qquad \text{(Eqn. 1.11)}$$

The total energy of the electron, E, therefore is obtained:

$$E = U + T = -\frac{Ze^2}{4\pi \varepsilon_0 r} + \frac{1}{2}\frac{Ze^2}{(4\pi \varepsilon_0)\, r} = -\frac{1}{2}\frac{Ze^2}{(4\pi \varepsilon_0)\, r} \qquad \text{(Eqn. 1.12)}$$

The total energy is negative - a general characteristic of bound orbits. This equation also tells us that if we know the radius of the orbit (r) we can calculate the energy E of the electron.

Substituting the expression for r_n (Eqn. 1.9) into Eqn. 1.12, we obtain for the energy:

$$E = -\frac{1}{2}\frac{Ze^2}{4\pi\varepsilon_0} \times \frac{1}{n^2}\frac{Zme^2}{4\pi\varepsilon_0\hbar^2}$$

or

$$E = -\frac{1}{2}\left(\frac{Z\,e^2}{4\pi\,\varepsilon_0\,\hbar}\right)^2 \frac{m}{n^2} = Z^2\,\frac{E_1}{n^2} \qquad \text{(Eqn. 1.13)}$$

where

$$E_1 = -\frac{me^4}{32\,\pi^2\,\varepsilon_0^2\,\hbar^2} = -13.58 \text{ eV}$$

is the energy of the electron in its lowest or "ground" state in the hydrogen atom.

Assumption 4: Radiation is emitted only from transitions between the discrete energy levels

The second Bohr postulate now defines the nature of the spectrum produced from these energy levels. This postulate declares that when an electron makes a transition from a higher to a lower energy level, a single photon will be emitted. This photon will have an energy equal to the difference in energy of the two levels. Similarly, a photon can only be absorbed if the energy of the photon corresponds to the difference in energy of the initial and final states. Schematically, we have:

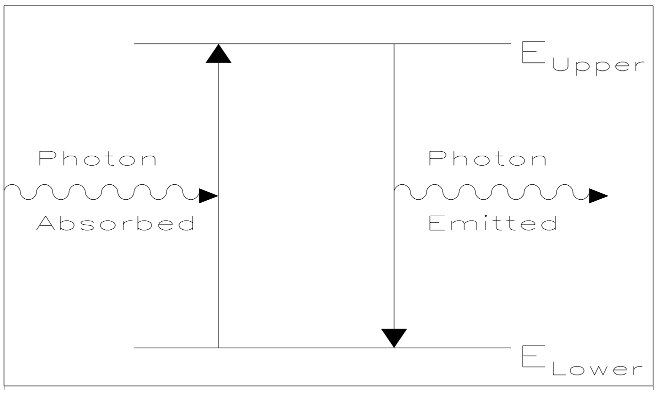

Figure 1.4 Second Bohr Postulate - photons produced/destroyed by discrete transition in energy.

Figure 1.5 illustrates the energy levels for the Bohr model of the hydrogen atom, and the associated "potential well" which is formed for the trapped orbits. We find that the ionization energy, the energy necessary to remove the electron from its "well", is 13.58 eV. If the electron gains somewhat less energy, it may move up to an "excited state", where n > 1.

For example, if it gains

$$13.58 - 3.39 = 10.19 \text{ eV}$$

it will move up to the n = 2 level.

Dropping down from n = 2 to n = 1, it will emit a photon of 10.19 eV energy, at a wavelength

$$\lambda = \frac{hc}{\Delta E} = 121.8 \text{ nm} = 1218 \text{ Å}$$

If ΔE is expressed in electron-volts (eV), which it usually is, then the constant "hc" in the numerator can be written as:

$$hc = 4.14 \times 10^{-15} \bullet 3 \times 10^{8} = 1.24 \times 10^{-6} \text{ (eV m)}$$

and thus the wavelength λ (in meters) is given by:

$$\lambda \text{ (m)} = \frac{1.24 \times 10^{-6}}{\Delta E \text{ (eV)}} \quad \text{or} \quad \lambda \text{ (nm)} = \frac{1240}{\Delta E \text{ (eV)}}$$

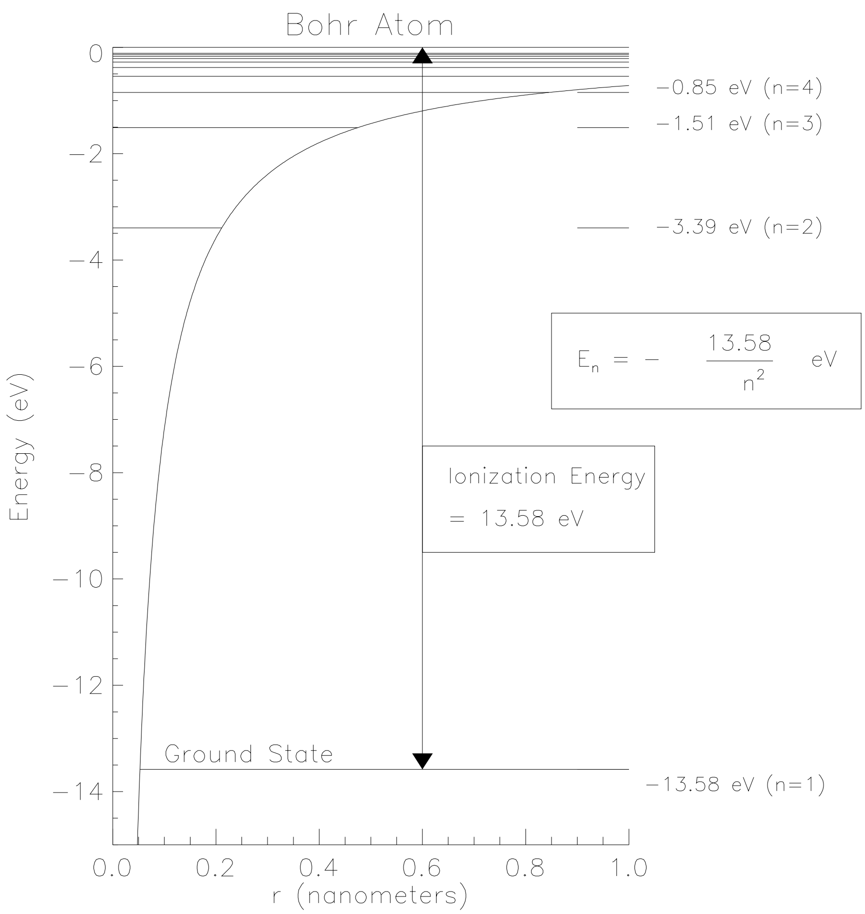

Figure 1.5 Energy levels for the hydrogen atom, according to the Bohr model.

In general, transitions will occur between different energy levels, resulting in a wide spectrum of discrete spectral lines. This is further illustrated in Figure 1.6. Transitions from (or to) the n = 1 energy level (the ground state) are called the Lyman series. The n = 2 to n = 1 transitions is the Lyman alpha (α) transition. This ultraviolet (UV) emission is one of the primary spectral (emission) lines of the sun's upper atmosphere. (See also, Figure 1.2) The emission (or absorption) lines in the visible portion of the sun's spectrum are the Balmer series, transitions from n > 2 to n = 2. Higher order series are of less importance for our purposes.

The Bohr model is successful in predicting the observed energy levels for one-electron atoms. It is useful for illustrating the quantum nature of the atom, and the associated energy levels. The model was ultimately replaced by the solution of the Schrodinger equation, and a more general form of quantum mechanics.

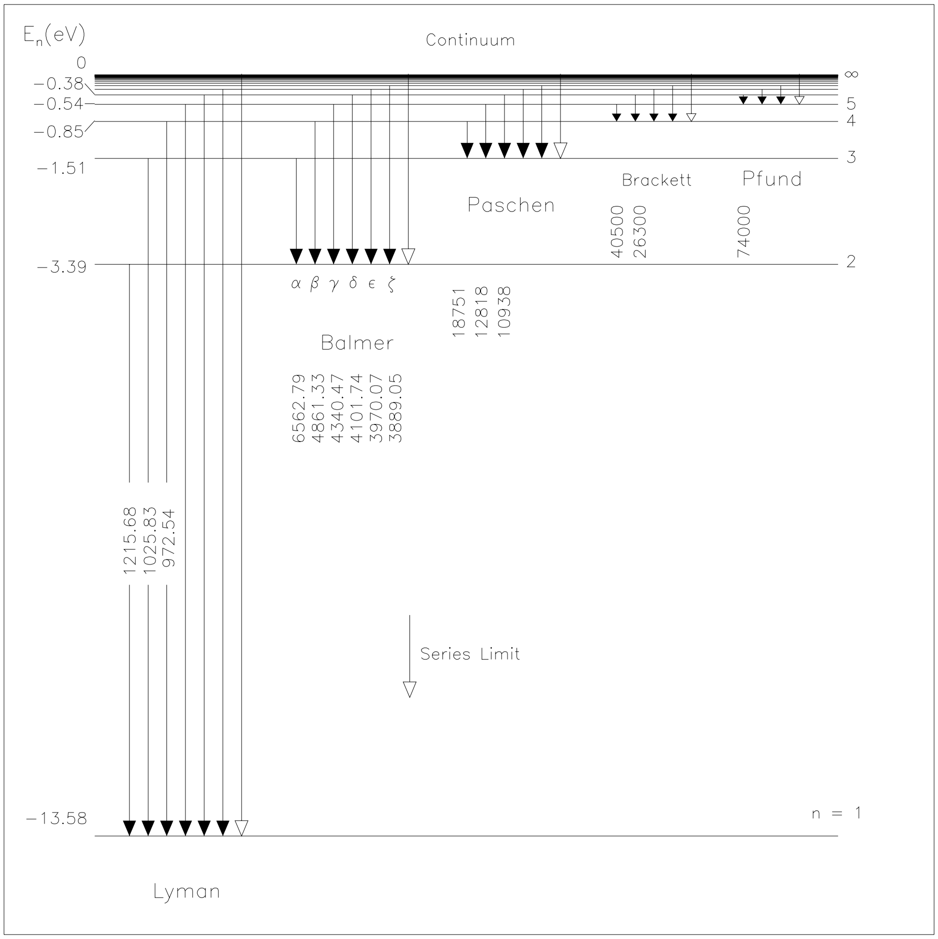

Figure 1.6. The energy level diagram of the hydrogen atom, showing the possible transitions corresponding to the different series. The numbers along the transitions are wavelengths. (Wavelengths are in units of Å, where 1 nm = 10 A)

Adapted from : Fundamentals of Atomic Physics, Atam P. Arya, p264, 1971.

2 Black body radiation

Black Body Radiation is emitted by hot solids, liquids or dense gases and has a continuous distribution of radiated wavelength as shown in Figure 1.7. The shape and height of the curve in Figure 1.7 is determined by two important constants: the emissivity ε and the temperature T.

The emissivity of a surface is a measure of the efficiency with which the surface absorbs (or radiates) energy and lies between 0 (for a perfect reflector) and 1 (for a perfect absorber). A body which has ε = 1 is called a black body and we shall assume that the radiators we deal with in space behave like black bodies.

Temperature is the most important parameter for black bodies and the amount of power radiated per unit surface area depends on temperature only. In Figure 1.7 we see a plot of radiated power/unit area/unit wavelength interval vs. wavelength known as Planck's Radiation Law. For our purposes we are particularly interested in two aspects: The total power radiated which is represented by the area under the curve and the wavelength at which the curve peaks, λ_{max}.

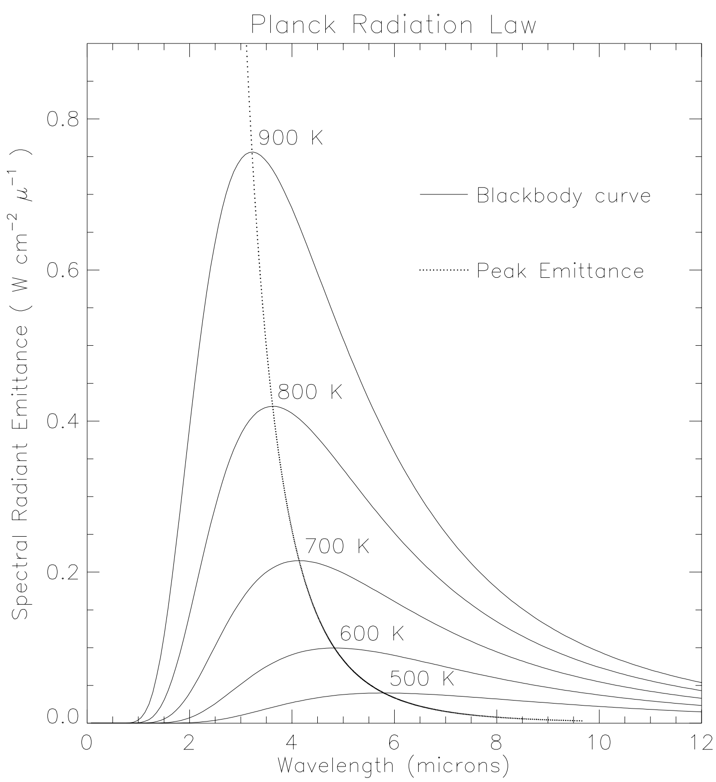

Figure 1.7 Black body radiation spectrum

$$U(\lambda) = \frac{8\pi hc}{\lambda^5}\frac{1}{e^{\left(\frac{hc}{\lambda kT}\right)}-1}$$

$$= \frac{8\pi (kT)^5}{(hc)^4}\frac{\left(\frac{hc}{\lambda kT}\right)^5}{e^{\left(\frac{hc}{\lambda kT}\right)}-1}$$

Student exercise: check long wavelength behavior - use the fact that for small x:

$e^x - 1 \approx x$

to get rid of the exponential term in the denominator.

Also, which term dominates U for small wavelength ?

The power radiated (integrated over all wavelengths) is given by

$$R = \sigma \varepsilon T^4 \left(\mathrm{Watts} / \mathrm{m}^2 \right) \quad \text{(Stefan Boltzmann Law)} \qquad \text{(Eqn. 1.14)}$$

where R = Power radiated / m^2

ε = Emissivity (taken as unity for black body)

$\sigma = 5.67 \times 10^{-8} \left(\mathrm{W} / \mathrm{m}^2\, \mathrm{K}^4 \right)$ (Stefan's Constant)

T = Temperature of the radiator (in K)

The wavelength at which the peak in radiation occurs is given by Wien's Displacement Law:

$$\lambda_{max} = a / T \qquad \text{(Eqn. 1.15)}$$

for a given temperature T. The constant "a" has the value

$$a = 2.898 \times 10^{-3} \; (\mathrm{m\,K})$$

which gives λ_{max} in meters if T is K.

Example:

Assume that the sun radiates like a blackbody, which is not a bad assumption, though we must choose two slightly different temperatures to match the observed quantities.

(a) Find the wavelength at which this radiation peaks, $\lambda_{max.}$ The solar spectral shape in the visible is best matched by a temperature of ~ 6000 K.
(b) Find the total power radiated by the sun. The Stefan-Boltzmann law is best served by an "effective temperature" of ~ 5800 K.

Solution:

$$\lambda_{max} = \frac{a}{T} = \frac{2.898 \times 10^{-3} \; (\mathrm{m/K})}{6000 \; \mathrm{K}} = 4.83 \times 10^{-7} \; \mathrm{m}$$

The spectrum peaks at ~500 nm, as illustrated below in Figure 1.8.

Next we can calculate R, the power emitted per-square-meter of surface. We use: $R = \sigma \varepsilon T^4$ and we assume that ε = 1 (Black Body). Evaluating, we get:

$$R = 5.67 \times 10^{-8} \bullet 1 \bullet 5800^4 = 6.42 \times 10^7 \; \frac{\mathrm{Watts}}{\mathrm{meter}^2}$$

To find the total solar power output we must multiply by the solar surface area , $S_\odot = 4 \pi R_\odot^2$, where $R_\odot = 6.96 \times 10^8$ m is the mean radius of the sun. Hence the total solar power output is:

$$P_\odot = R\,(4\,\pi\,R_\odot^2) = 4\,\pi\,(6.96 \times 10^8)^2 \times (6.42 \times 10^7)$$

$$P_\odot = 3.91 \times 10^{26}\ \mathrm{W}$$

See Kenneth Phillips, *Guide to the Sun*, Cambridge Press, 1992., pages 83-84) The sun's spectrum is shown in Figure 1.8, with the spectrum of a 5,800 K black body superimposed.

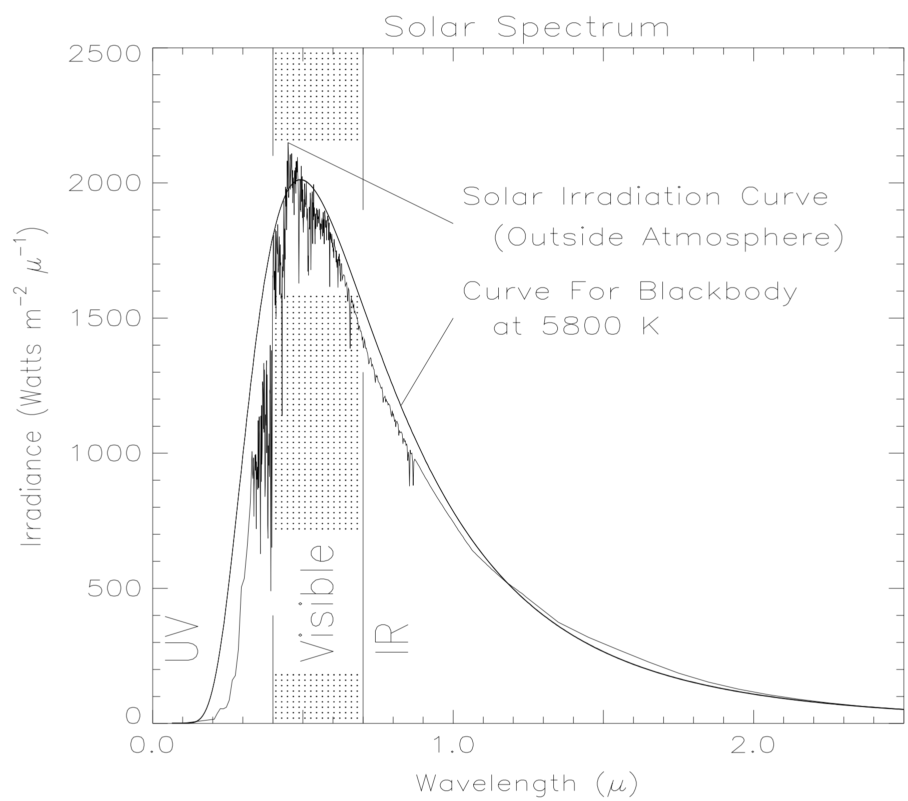

Figure 1.8 - The solar spectrum, based on the spectrum of Neckel and Labs “The solar radiation between 3300 and 12500 Angstrom, Solar Physics, 90, 205-258, 1984. Data file courtesy of Bo-Cai Gao, NRL. The peak occurs at about 460 nm (blue).

3 X-rays - Brehmstrahlung

An important source of x-rays in space comes from the sudden deceleration of high speed electrons. This is also a common process by which x-rays are generated in the laboratory (or the dentist's office) . It is generally true that when an electron is slowed down by collisions or deflected by an electric or magnetic field it will radiate electromagnetic energy (photons). The production mechanism most commonly found in space involves a (mono-)energetic beam of electrons which are decelerated by impact with a material surface or a dense gas (or plasma). The electron beam will produce radiation with a continuous spectrum of frequencies up to a maximum frequency, f_{max}, given by: $E = hf_{max}$. Here, E is the original electron energy, or the energy of the electron beam in the case being presented here. Figure 1.9 shows the type of spectrum produced in the laboratory, in this case exposing a tungsten target to electron beams at various energies. Note that in this figure, the spectra are plotted vs. wavelength, not frequency. Hence, the cutoff at f_{max} corresponds to a spectrum which terminates at a minimum wavelength, λ_{min}, below which no x-rays are generated.

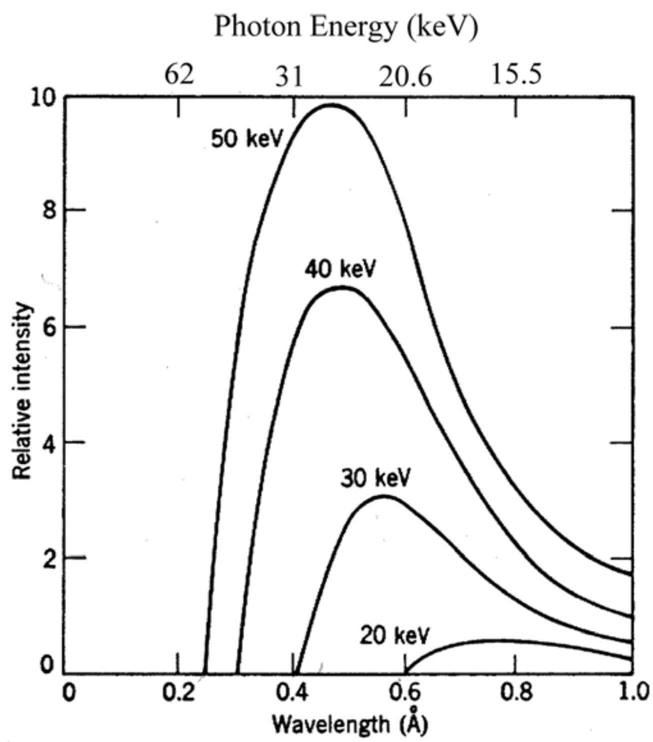

Figure 1.9 Spectrum for x-rays generated by an electron beam (Brehmstrahlung). From: Eisberg and Resnick, Quantum Physics, (fig 2-10), 1985.

C Kinetic Theory

Before proceeding into the concepts of plasma physics, a few elements of the classical kinetic theory of gases need to be established - primarily the relationship between the temperature of an ensemble (collection) of atoms, and the kinetic energy of individual atoms.

1 Temperature and Kinetic Energy

Kinetic theory says that the mean kinetic energy of a molecule in a gas is related to the temperature of the gas by the relation:

$$\frac{1}{2}mv_{rms}^2 = \frac{3}{2}kT \qquad \text{(Eqn. 1.16)}$$

where $v_{rms} = \sqrt{\left\langle v^2 \right\rangle}$.

Here, k is the Boltzmann constant, $k = 1.38 \times 10^{-23}\ {}^{J}\!/\!_{K}$, and the brackets $\langle\ \rangle$ indicate an average over the volume of particles at hand. The velocity (v_{rms}) is the root-mean-square velocity. The energy described by this relation is again the average kinetic energy of an atom or molecule. These particles are colliding with each other continually, and these collisions can excite electronic transitions in the atoms or molecules. If the collisions are sufficiently energetic, an electron can be knocked free, ionizing the particle. For a hydrogen atom, the energy required is $E \approx 13.6$ eV. The temperature associated with this (average kinetic) energy is therefore:

$$kT \approx 13.6\ \text{eV} \bullet 1.6 \times 10^{-19} \left(\frac{J}{eV}\right) \quad \text{or} \quad T \approx \frac{13.6 \bullet 1.6 \times 10^{-19}}{1.38 \times 10^{-23}} = 1.6 \times 10^5\ K$$

We can see that the temperature required for "thermal" ionization of hydrogen atoms is of such a magnitude as is normally only found in regions of very high temperatures such as the solar corona. (Note that the factor of ${}^{3}\!/\!_{2}$ has been cheerfully ignored - partly to avoid a false implication of accuracy in the above estimate).

2 Maxwellian Distributions

In equilibrium distributions of gas molecules, where concepts such as temperature make sense, it is typically found (and often assumed) that the distribution of atomic or molecular velocities follow the well-known Maxwellian distribution. This will occasionally be referred to in the following material, and it is helpful to illustrate the distribution here.

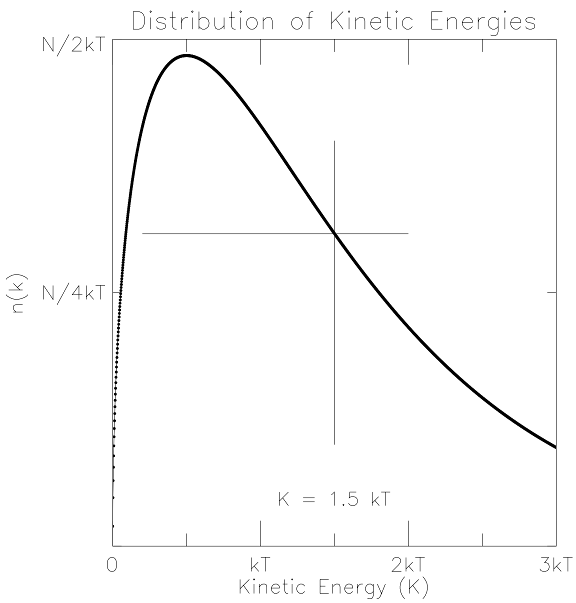

Figure 1.10 Maxwellian Distribution

$$n(K) = \frac{2N\sqrt{K}\,e^{-K/kT}}{\sqrt{\pi}\,(kT)^{3/2}} \quad \text{(Eqn. 1.17)}$$

gives the number of particles, n, which have a kinetic energy, K, for a gas volume with a total number of particles (e.g. molecules), N, at a temperature, T. Here, this equation is used in the well defined, and classic, integral:

$$N = \int_{K=0}^{\infty} \frac{2N\sqrt{K}\,e^{-K/kT}}{\sqrt{\pi}\,(kT)^{3/2}}\,dK$$

in order to determine the total number of particles, N, in a given volume. Useful questions: Where does the distribution peak What is the value of n(K) at the maximum? What is the value at K = 1.5 kT?

D Plasmas

1 Phases of matter

It is estimated that most of the matter in the universe exists in the plasma state and that solid objects such as the planets are only minor specks of dust floating in this plasma space. Plasmas are distinguished from the traditional three states of matter primarily by the temperature range at which most plasmas are found. Figure 1.11 illustrates the range of temperatures for which matter generally exists in a plasma state.

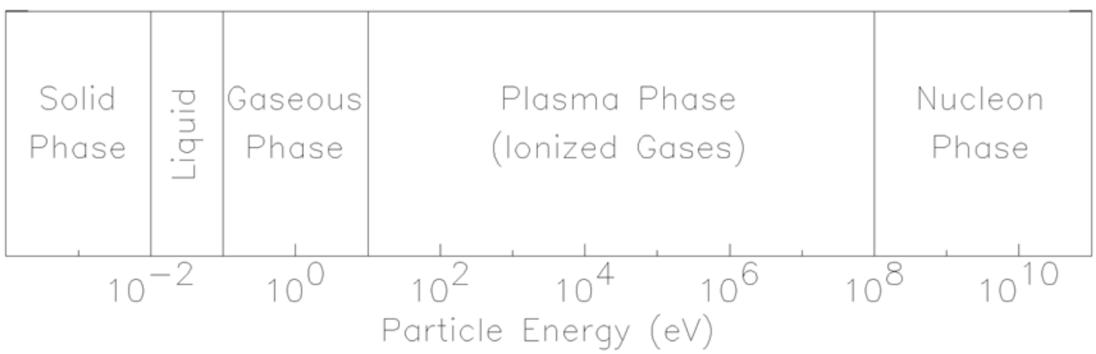

Figure 1.11 Phases of matter as a function of average particle energy (temperature)

A plasma is a quasi-neutral gas of charged and neutral particles which exhibits collective behavior. The charged particles are typically (positively charged) ions and electrons, and the net charge in a region is typically very small. Plasmas typically consist of only partially ionized gas. For example, the gas found at 1-5 earth radii (altitude) is ~50% ionized. The definition of a plasma can be quantified, to an extent, by the usage of two quantities defined here, the Debye Length (λ_D) and the plasma frequency (ω_p). We begin by developing the Debye Length.

2 Plasma Parameters

Debye Shielding length (λ_D)

One important characteristic of a plasma is the shielding of electric fields produced by the ensemble of particles. The concept of the Debye length (λ_D) also allows us to help define when we have a plasma.

Conceptually, we begin with an object in a plasma (e.g. a probe, or satellite) with a charge on it. In a vacuum we would have the Coulomb potential

$$\phi(r) = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r} \qquad \text{(Eqn. 1.18)}$$

If we immerse this same point charge +Q in a plasma it will attract some of the negative charges and repel the positive charges leading to a partial shielding of the charge at distant points. The potential at a point distant r from our charge +Q now becomes:

$$\phi(r) = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r}\, e^{-r/\lambda_D} \qquad \text{(Eqn. 1.19)}$$

where $$\lambda_D\,(\text{meters}) = \left(\frac{\varepsilon_0\,kT}{e^2\,n_e}\right)^{1/2} \qquad \text{(Eqn. 1.20)}$$

is the Debye length, and

n_e = electron density (m^{-3})

k = Boltzmann constant = 1.381 x 10^{-23} (J/ K) T = Temperature (K)

e = electronic charge = 1.602 x 10^{-19} (Coulombs) ε_o = 8.854 x 10^{-12} (Farads/m)

The two important parameters are the temperature of the electrons T (or equivalently their average kinetic energy kT) and the electron density n_e. For quick calculations we may write the Debye length (λ_D) as:

$$\lambda_D\,(m) = 69\left(\frac{T}{n_e}\right)^{1/2} \text{ where T is in K, } n_e \text{ is in electrons/m}^3 \qquad \text{(Eqn. 1.21)}$$

$$\lambda_D\,(m) = 7430\left(\frac{kT}{n_e}\right)^{1/2} \text{ where kT is in eV, } n_e \text{ is in electrons/m}^3 \qquad \text{(Eqn. 1.22)}$$

Note the inverse dependence on density - higher density plasmas provide more shielding. (Also note, it is common in space physics to use electron Volts (eV) as a unit of temperature, as well as a non-mks unit of energy.)

Example:

n = 5×10^{10} electrons/m^3; T = 22,000 K	n = 5×10^4 electrons/cm^3, (Note units - must convert to electrons/m^3); T = 2eV
$\lambda_D\,(m) = \left(\frac{\varepsilon_0\,kT}{e^2\,n_e}\right)^{1/2}$ $= \left(\frac{8.85\times10^{-12}\bullet\,1.38\times10^{-23}\,\bullet\,22\times10^3}{\left(1.6\times10^{-19}\right)^2\bullet\,5\times10^{10}}\right)^{1/2}$ = 0.0458 meters = 4.6 cm	$\lambda_D\,(m) = \left(\frac{\varepsilon_0\,kT}{e^2\,n_e}\right)^{1/2}$ $= \left(\frac{8.85\times10^{-12}\bullet\,1.6\times10^{-19}\bullet\,2}{\left(1.6\times10^{-19}\right)^2\bullet\left(5\times10^4\bullet10^6\right)}\right)^{1/2}$ = 0.047 meters = 4.7 cm
$\lambda_D\,(m) = 69\left(\frac{T}{n_e}\right)^{1/2} = 69\left(\frac{22\times10^3}{5\times10^{10}}\right)^{1/2}$ = 0.0458 meter = 4.6 cm	$\lambda_D\,(m) = 7430\left(\frac{kT}{n_e}\right)^{1/2} = 7430\left(\frac{2}{5\times10^4\bullet10^6}\right)^{1/2}$ = 0.047 meters = 4.7 cm

There is a reason why the answers in the two columns are (almost) the same - the 22,000 K temperature converts to 2 eV.

Plasma Frequency (ω_p)

The collective behavior of plasmas is, in many ways, exhibited most strongly in the enormous variety of electromagnetic waves which occur in ionized media. The most prominent of these wave modes is the plasma oscillation, which is a result of the oscillation of the electrons about the (relatively massive and immobile) ions. Much as in the case of a mass on a spring, the mass of the electrons determines the characteristic frequency. The total mass (per unit volume) is of course proportional to the electron density. The plasma frequency is defined as:

$$\omega_p = \left(\frac{n_e \, e^2}{\varepsilon_o \, m_e} \right)^{\frac{1}{2}} \qquad \text{(Eqn. 1.23)}$$

If the various constants are plugged in, we obtain:

$$\omega_p \text{ (radians / second)} = 56 \sqrt{n_e (\text{electrons/meter}^3)} .$$

In Figure 1.12 the ranges of several plasmas are shown as functions of the value of the Debye shielding length. Recall that the conversion from frequency in radians/second to Hertz is given by:

$$f_p \text{ (Hz)} = \frac{\omega_p (\text{radians / second})}{2\pi}$$

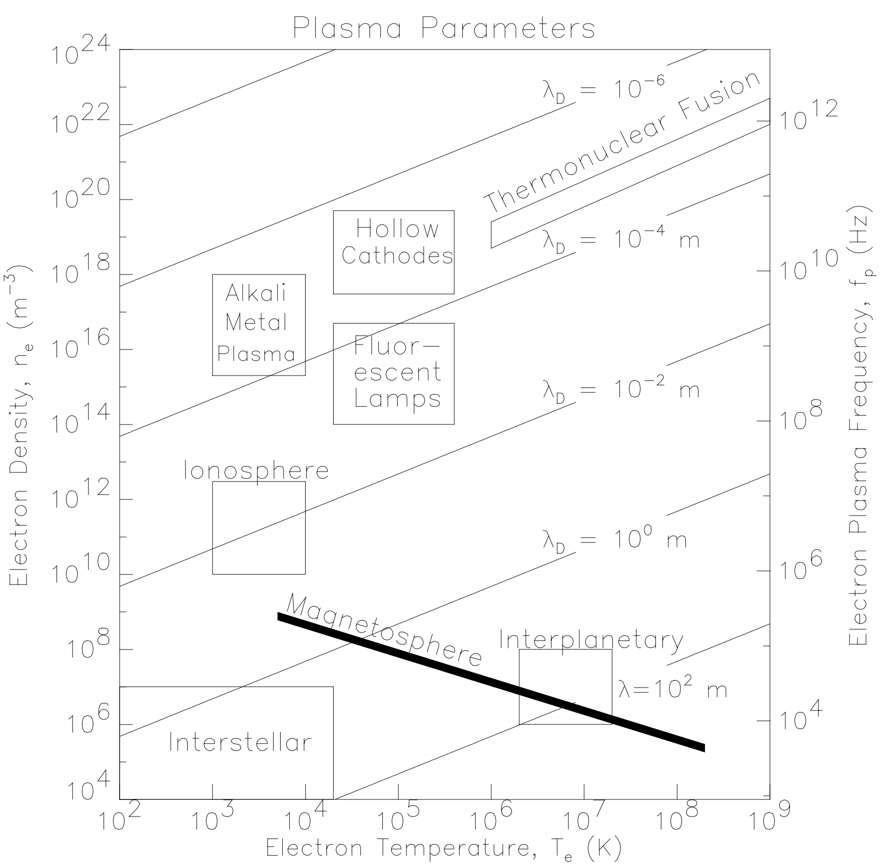

Figure 1.12 - Plasma parameters and Debye shielding length

3 Definition of Plasma

With the above two quantities defined, we are now able to consider the criteria which define the existence of a plasma.

1. The typical linear dimensions of the plasma (e.g. L) must be much larger than the characteristic plasma dimension, λ_D, the distance scale over which electric fields penetrate into a plasma, e.g.:

$$\lambda_D \ll L \qquad \text{(Eqn. 1.24)}$$

The characteristic scale size for variations in the ionosphere or higher altitude space regimes is many kilometers, and we see that the first criterion is easily met for the ionized particle distributions found in space.

2. The number of charges (N_D) in a sphere of radius λ_D must be very much larger than 1,

$$N_D = \left(\frac{4}{3}\pi\lambda_D^3\right)(n_e) = 1.38\times10^6 \frac{T^{3/2}}{n_e^{1/2}} \gg 1 \qquad \text{(Eqn. 1.25).}$$

3. The plasma frequency must be larger than the frequency of collisions. This can be written as $\omega_p \gg \nu_c$, where ν_c is the collision frequency. If this criterion is not met, there may be a high degree of ionization, but collective (wave) phenomena cannot occur.

Example:

Consider whether the terrestrial ionosphere can be considered to be a plasma by our criteria.

First condition: Typical experimental data for the ionosphere at an altitude of 90 km, mid-latitude, daylight are: $T \sim 800°$ K and $n_e = 10^{11}$ electrons / m^3

$$\lambda_D = 69 \left(\frac{T}{n}\right)^{1/2} = 6\times10^{-3}\ \text{m} = 6\ \text{mm}$$

Since the typical dimensions of the ionosphere are on the order of 100 km or more, the condition that the system be much larger than the Debye length is certainly satisfied

Second condition: $N_D \gg 1$.

$$N_D = 1.38\times10^6 \frac{T^{3/2}}{n_e^{1/2}} = 1.38\times10^6 \frac{800^{3/2}}{\left(10^{11}\right)^{1/2}}$$

$$N_D = 9.87 \times 10^4 \approx 10^5$$

so the second condition is also satisfied.

Third condition: This criterion requires the additional piece of information, that of the collision frequency (ν_c) at 90 km, which has a nominal value of 1.3×10^5 Hz. Comparing to the plasma frequency, we find $\omega_p = 56\sqrt{8.2 \times 10^9} = 5.1 \times 10^6$ (radians/second). The third criterion is satisfied reasonably well, but in fact collisional processes need to be considered carefully when studying ionospheric processes. Collisional frequencies drop rapidly with increasing altitude.

---

4 Plasma Theories and Approximations

The complete description of a plasma must include the motion of charged particles in time dependent electric and magnetic fields both applied and self generated. Kinetic Theory and the Boltzmann Transport Equation can be used to formulate this problem, but in practice the solutions are so difficult that little use is made of these equations for the actual solutions of plasma problems.

There exists however two approximations which apply in limiting situations and which are more manageable:

(a) The magneto-hydrodynamic approximation: When the particle density is high enough and collisions between the particles are frequent enough that we may define a thermodynamic temperature and pressure then we may treat the plasma as a conducting fluid. A combination of Maxwell's and fluid dynamic equations can be used to calculate such effects as plasma oscillations, hydromagnetic waves, etc. These equations are still quite complex but with certain approximations they can be solved. The concept of Debye shielding, described briefly above, arises from the hydrodynamic approximation.

(b) Single Particle Limit: When collisions between particles become very infrequent we can use the other limiting approximation, namely orbit theory. Here we assume that individual charges move in orbits determined by the applied electric and magnetic fields and that the fields generated by the moving charges themselves are small and can be neglected.

Both of these approaches are used in the analysis of plasma phenomena in space. We shall start with a brief review of electric and magnetic fields, and the forces they exert on electric charges. The single particle limit (orbit theory) is based directly on these concepts.

5 Orbit theory - Motion of Charged Particles

Lorentz Force

The electric field **E** is defined as the force exerted on a small positive test charge q_0 placed at the point in question. This is usually written as

$$\vec{E} = \lim_{q_o \to 0} \frac{\vec{F}_E}{q_o} \qquad \text{(Eqn. 1.26)}$$

(q_0 is positive and small enough so its effect on the other charges is negligible)

The units of electric field are: $\dfrac{\text{Newtons}}{\text{Coulomb}} = \dfrac{\text{Volts}}{\text{meter}}$

The magnetic field of induction $\vec{B}$ is defined by magnetic force $\vec{F}_B$ which a particle carrying a charge q will experience when it moves with velocity $\vec{v}$ at the point in question. Thus the defining equation for $\vec{B}$ may be written as

$$\vec{F}_B = q\left(\vec{v} \times \vec{B}\right) \qquad \text{(Eqn. 1.27)}$$

The unit of $\vec{B}$ is the Tesla, obtained directly from this equation.

Thus the force experienced by a charged particle moving in a combined electric ($\vec{E}$) and magnetic ($\vec{B}$) field is the so called Lorentz force given by

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) \qquad \text{(Eqn. 1.28)}$$

Motion of Charged Particles in Simple $\vec{E}$ and $\vec{B}$ Fields

Let us consider a couple of simple examples of a particle of constant mass m and charge q (can be + or -) moving under the influence of electric or magnetic forces. The actual motions can then (often) be understood as a combination of the motions defined below.

Example 1: B=0, Uniform E $\neq$0

Assume $\vec{B} = 0$ and $\vec{E}$ is uniform in space and constant in time.
The force on our particle is:

$$\vec{F} = q\vec{E}$$

and from Newton's Second Law

$$\vec{F} = m\vec{a} = q\vec{E} \quad \Rightarrow \quad \vec{a} = \frac{q}{m}\vec{E} = \text{a constant}$$

We thus have motion exactly analogous to motion under constant gravitational acceleration $\vec{g}$ but with one major difference:

These accelerations due to the $\vec{E}$ field can be along $\vec{E}$ (if q > 0), or along - $\vec{E}$ (if q < 0). Thus electrons and negative ions will accelerate in the - $\vec{E}$ direction, while positive ions will accelerate in the direction of $\vec{E}$.

Example 2: E=0, Uniform B≠0

Assume $\vec{E} = 0$ and $\vec{B}$ is uniform in space and constant in time. The motion will now depend on the direction of the initial velocity $\vec{v}_0$.

Special case 1: ($\vec{v}_0 \parallel \vec{B}$)

The initial velocity vector is parallel to the magnetic field. ($\vec{v}_0 \parallel \vec{B}$)

$\vec{F}_B = q\left(\vec{v} \times \vec{B}\right) = 0$

The particle will continue with its initial velocity $\vec{v}_0$.

Special case 2: ($\vec{v}_0 \perp \vec{B}$)

The initial velocity vector is perpendicular to the magnetic field. ($\vec{v}_0 \perp \vec{B}$). The magnitude of the deflecting force is now a constant $F_B = qv_0 B$ and always perpendicular to the instantaneous velocity vector, whose magnitude is also constant. This force will result in circular motion with radius r_c given by

$$r_c = \frac{mv_0}{qB} \qquad \text{(Eqn. 1.29)}$$

This quantity is known as the cyclotron or gyration radius and plays an important role in much of the material to follow. Equally important are the period T of the motion and its inverse, the gyrofrequency.

$$T = \frac{2\,\pi\, r_c}{v_0} = \frac{2\,\pi\, mv_0}{qB\, v_0} = \frac{2\,\pi\, m}{qB} \quad \text{(seconds)}$$

or

$$f_c = \frac{1}{T} = \frac{qB}{2\,\pi m} \text{ (revolutions / second)} \Rightarrow \qquad \text{(Eqn. 1.30)}$$

$$\omega_c = 2\,\pi f_c = \frac{qB}{m} \text{ (radians / second)}$$

The gyrofrequency is independent of the speed and hence the energy of the gyrating particle. In a given field it depends only on the charge to mass ratio of the particle. Consider a uniform field B pointing out of the paper. Particles of opposite charge will gyrate with opposite direction as shown. Note that the particle motion is diamagnetic - the magnetic field created by the orbiting charged particle opposes the applied field.

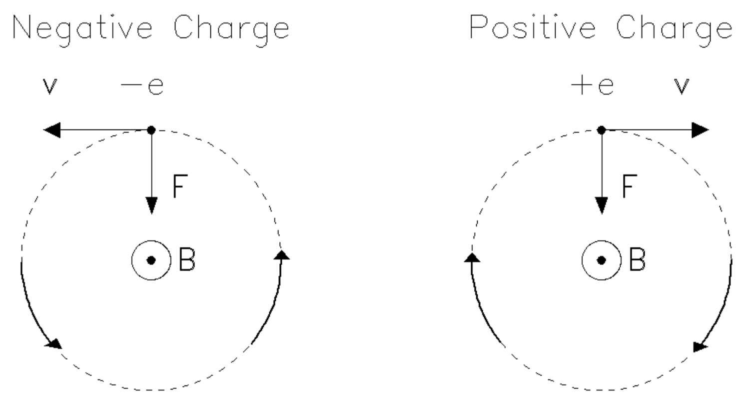

Figure 1.13 Charged particle motion in a uniform magnetic field. The magnetic field is 'up', out of the page.

General Case: Arbitrary direction of $\vec{v}_0$

If the initial velocity vector is neither parallel nor perpendicular to the $\vec{B}$ field the trajectory will be a helix of constant pitch and radius which results from the superposition of circular motion in the perpendicular plane and uniform transitional motion along the field direction. Hence the axis of the helix will be parallel to $\vec{B}$.

Particle Drifts: E≠0, B≠0

In the first two examples above, one of the two terms (E, B) is zero. If both are non-zero, a peculiar result is obtained. It is found that for any force $\vec{F}$ applied to a particle, in combination with the magnetic force, the equation of motion produces a motion for the center of the orbit termed a "drift velocity" given by:

$$\vec{v}_D = \frac{1}{q}\frac{\vec{F}_\perp \times \vec{B}}{B^2}. \qquad \text{(Eqn. 1.31)}$$

$\vec{F}_\perp$ is the component of the force perpendicular to the direction of $\vec{B}$. (Realistically, this is almost always the electrical force, F = qE) We shall now consider two examples of Eqn. 1.31:

Example 1:

Suppose the additional force $\vec{F}$ is the gravitational force $m\vec{g}$ and let us assume that it is perpendicular to $\vec{B}$ (This would be the case at the geomagnetic equator). Then:

$$\vec{\mathbf{v}}_{\mathbf{D}} = \frac{1}{q}\frac{m\vec{\mathbf{g}} \times \vec{\mathbf{B}}}{B^2}$$

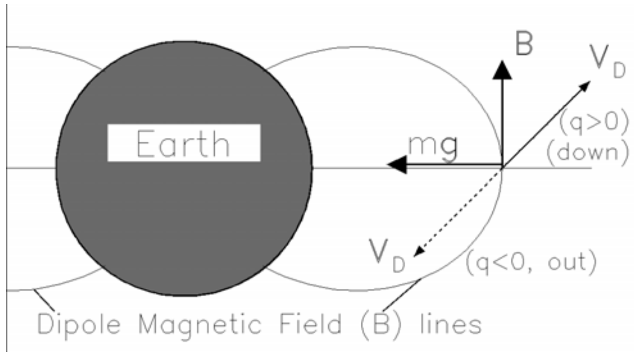

Figure 1.14 Drift directions due to gravity. Positively charged particles drift "down" or into the page, negatively charged particles drift "up" or out of the page.

It can be seen that positive and negative charges will drift in opposite directions. Thus protons will precess in an easterly directions and electrons in a westerly direction forming ring currents around the earth. The magnitude of the drift velocities for this case are rather small due to the small mass of both the proton and electron.

At an altitude of one earth radius (6400 km); the geomagnetic field has a magnitude of 3.75×10^{-6} T (3.75 μT). The charge of both proton and electron has a magnitude of 1.6×10^{-19} Coulombs. The proton mass is 1.67×10^{-27} kg and the electron mass is 9.1×10^{-31} kg.

Putting all these numbers into the formula we obtain

Proton drift velocity $\approx 3 \times 10^{-2}$ m/sec
Electron drift velocity $\approx 1.5 \times 10^{-5}$ m/sec

Note however that positive charges moving eastward and negative charges moving westward both contribute to a net eastward current, so that a net current is generated.

Example 2:

Suppose that we now place our moving charge in a crossed electric and magnetic field so that our force $\vec{F}$ in equation 1.31 is given by: $\vec{F} = q\vec{E}$
and we again assume that $\vec{E}$ (and hence $\vec{F}$) is perpendicular to $\vec{B}$ as shown below

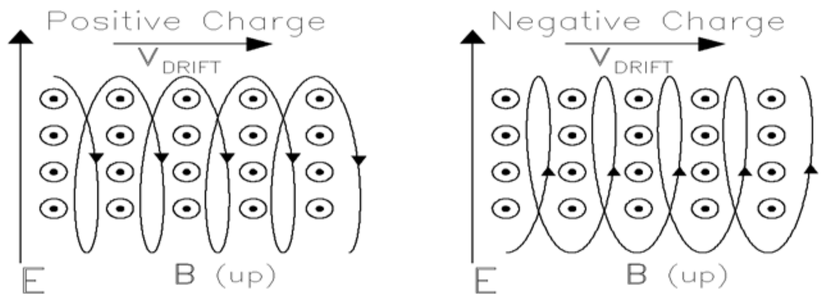

Figure 1.15 Drift directions due to an electric field

$$\vec{\mathbf{v}}_D = \frac{1}{q}\frac{q\,\vec{\mathbf{E}}\times\vec{\mathbf{B}}}{B^2} = \frac{\vec{\mathbf{E}}\times\vec{\mathbf{B}}}{B^2} \qquad \text{(Eqn. 1.32)}$$

Which says that <u>both</u> positive and negative charges drift in the same direction (to the right in our case). This happens because the force $\vec{F}$ is charge dependent and q cancels in the equation for $\vec{\mathbf{v}}_D$. Hence there is <u>no net</u> current in this case.

We can see why the drift is to the right in both cases. For positive charges (clockwise rotation) the energy (speed) is lower at the bottom and hence the radius of curvature is smaller. Near the top the particle has picked up energy from the electric field and hence its radius of curvature is now larger. The net effect is a displacement to the right.

For the negative charge the sense of rotation is reversed and the electric force acting on it is now downward. Combining these two effects again produces a drift toward the right as shown in the sketch. Note that the electric field induced drift does <u>not</u> result in a net current.

In our study of particle motion in the geomagnetic field we shall consider two more cases of particle drifts caused by gradients and curvature in the field itself.

A brief derivation of the drift equation

The drift equation can be derived in a fairly simple form for the Lorentz force. We make a few simplifying choices about directions, and then proceed. The magnetic field is taken to be in the **z** direction, the electric field in the **y** direction.
To begin:

$$\vec{F} = q\left(E_y\hat{y} + \vec{v}\times B_z\hat{z}\right) \qquad \text{(Eqn. 1.28)}$$

which has two (interesting) components, F_x and F_y.

$$\begin{aligned} F_x &= m\ddot{x} = 0 + q\, v_y\, B_z \\ F_y &= m\ddot{y} = qE_y - q\, v_x\, B_z \end{aligned} \qquad \text{(Eqn. 1.33)}$$

Note that the acceleration is just the derivative of the velocity:

$$\begin{aligned} m\dot{v}_x &= q\, v_y\, B_z \\ m\dot{v}_y &= qE_y - q\, v_x\, B_z \end{aligned} \qquad \text{(Eqn. 1.34)}$$

Now we know that in the absence of an electric field, the solutions to these two equations is just circular motion, the primary motion of a charged particle in a magnetic field.

$$E = 0 \Rightarrow v_x = v_o \cos\omega t;\; v_x = \mp v_o \sin\omega t \qquad \text{(Eqn. 1.35)}$$

where the top sign is for positively charged particles (protons), the lower for negatively charged particles (electrons). In fact, by being just a bit clever, we can make use of these solutions to solve the current problem. The solution comes by forming a new variable, v'_x which is comprised of the circular motion defined above, and a constant offset. Note that this constant offset has a derivative with respect to time that is zero.

$$v'_x = v_x - v_{o;}\;\; \dot{v}'_x = \dot{v}_x \qquad \text{(Eqn. 1.36)}$$

now we use this new variable in the above differential equation:

$$\begin{aligned} m\dot{v}'_x &= q\, v_y\, B_z \\ m\dot{v}_y &= qE_y - q\left(v'_x + v_o\right) B_z \end{aligned} \qquad \text{(Eqn. 1.37)}$$

if $E_y = v_o\, B_z$ or $v_o = \dfrac{E_y}{B_z}$ the new equations now reduce to:

$$\begin{aligned} m\dot{v}'_x &= q\, v_y\, B_z \\ m\dot{v}_y &= -\, q v'_x\, B_z \end{aligned} \qquad \text{(Eqn. 1.38)}$$

which are already know to have a solution which is simply uniform circular motion (Exercise for the student: show that the solutions given in Eqn. 1.35 solve Eqn. 1.38). Hence, v_x is simply

$$v_x = \mp v_o \sin\omega t + \frac{E_y}{B_z} \qquad \text{(Eqn. 1.39)}$$

This can be compared to Eqn. 1.32; the constant term is the drift velocity, $\vec{\mathbf{v}}_D = \dfrac{\vec{\mathbf{E}}\times\vec{\mathbf{B}}}{B^2}$

6 A few results from the magneto-hydrodynamic approximation

Magnetic Pressure

We shall now use the fluid approximation in which we assume that a plasma can be considered to be an electrically conducting fluid which moves under the influence of electrical, magnetic, gravitational and pressure gradient forces. Consider a streaming plasma which has a magnetic field B embedded within it. If the plasma is not accelerating or being constricted in its flow we can show that

$$P_K + \frac{B^2}{2\mu_o} = \text{constant} \qquad \text{(Eqn. 1.40)}$$

where $P_K =$ pressure due to particle motion $\left(N/m^2\right)$

(usually equal to the plasma pressure, nkT)

$$\frac{B^2}{2\mu_0} = \text{Magnetic energy density } \left(J/m^3\right)$$

$$= \text{Magnetic pressure } = P_M$$

and $\mu_o = 4\pi \times 10^{-7}$ (H/m), the permeability constant.

This equation says that in a region in which there is both plasma and magnetic field present we can consider the magnetic energy density to play the role of a magnetic pressure $P_{M.}$ (Dimensionally energy density and pressure are equivalent). The fact that the sum of kinetic pressure P_K and the magnetic pressure P_M is constant turns out to be of particular interest at the interface between plasma regions and field regions.

In regions where both stationary plasmas and magnetic fields are present it is customary to define a parameter β given by

$$\beta = \frac{\text{Kinetic Energy Density}}{\text{Magnetic Energy Density}} = \frac{\sum_i n_i \, kT_i}{B^2/2\mu_o} \qquad \text{(Eqn. 1.41)}$$

where n_i = particle density i_{th} species (m^{-3})
k = Boltzmann's Constant
T_i = Temperature of i_{th} species

Thus the numerator represents the kinetic energy density of the plasma and the denominator represents the magnetic energy density. The numerator is typically equal to $n\left(kT_{electron} + kT_{ion}\right)$, with the ions and electrons having a common density (quasi-neutrality), but not necessarily a common temperature.

Thus a high β plasma has a large kinetic energy density and the behavior of the system will be dominated by the particles. If on the other hand β is very small the magnetic field dominates and determines the behavior of the system.

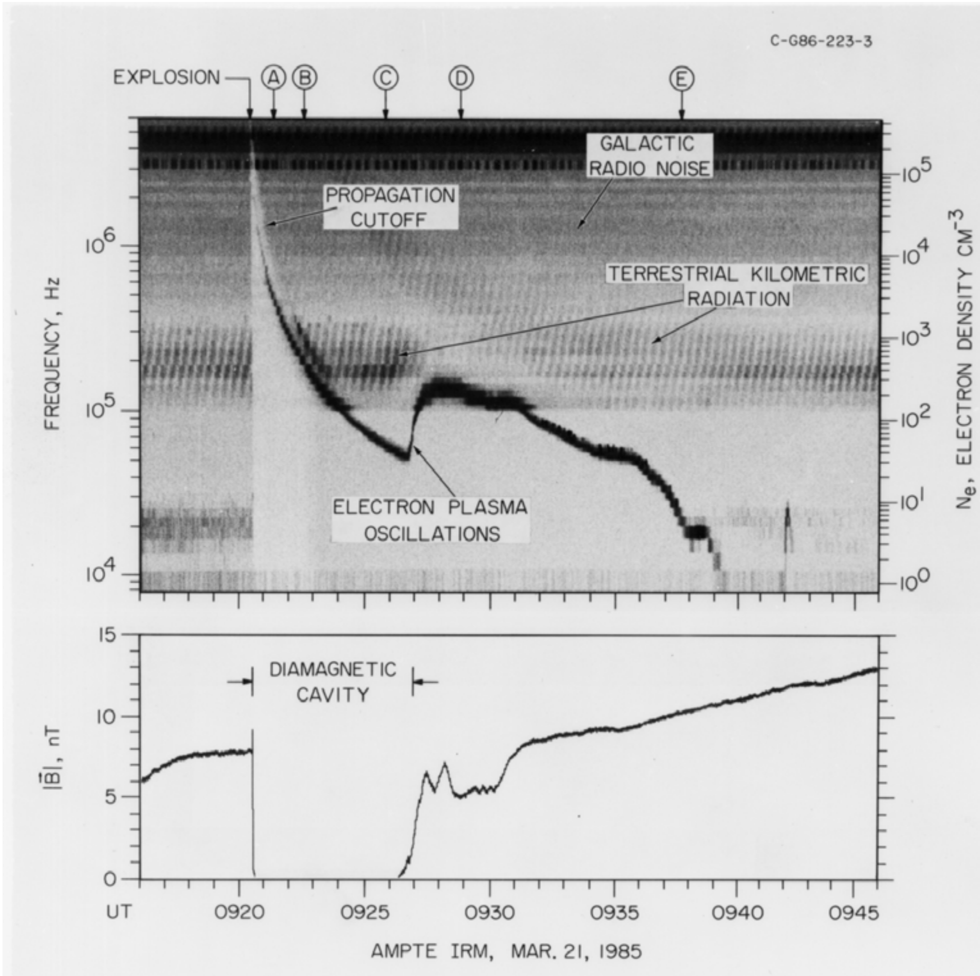

Figure 1.16 This image is from the paper: Observations and Theory of the AMPTE Magnetotail Barium Releases, P. A. Bernhardt et al, Journal of Geophysical Research, vol 92, page 5777, 1987. The figure was obtained from Dr. Don Gurnett, University of Iowa.

"Frozen-in" Magnetic Field Lines:

Consider a plasma which is embedded in a magnetic field. The field will in general be non-uniform as shown. If now either the plasma or the magnetic field configuration moves there will be an induced electromotive force generated within the plasma according to Faraday's Law

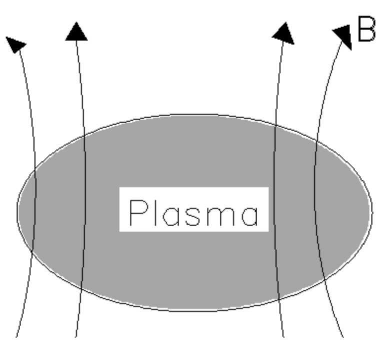

Figure 1.17

$$\mathcal{E} = -\frac{d\,\Phi_B}{dt} = -\int \frac{\partial \vec{\mathbf{B}}}{\partial t} \bullet d\vec{\mathbf{A}}$$
(Eqn. 1.42)

This electromotive force will generate an electric current within the plasma in such a direction that the magnetic field generated by the induced current opposes the change in magnetic flux which caused the electromotive force (Lenz's Law). If the conductivity of the plasma is high enough the induced currents and magnetic fields can become large enough to prevent any change in the external magnetic field. When these conditions apply we say that the field lines are "frozen" into the plasma and as the plasma moves the field lines will follow.

There exists a useful criterion which tells us when the "frozen-in" conditions are applicable. We define a quantity called the magnetic Reynolds number by

$$R_M = \mu_0 L v \sigma \quad \text{where} \qquad \text{(Eqn. 1.43)}$$

L = Approximate size of the plasma system (m)
v = velocity of fluid (m/sec)
σ = electrical conductivity of fluid (Siemens/m = (ohm - m)$^{-1}$)
μ_0 = $4\pi \times 10^{-7}$ (H/m)

R_M is a dimensionless quantity (pure number)

If $R_M >> 1$ the B field will be frozen into the fluid and will move with it.

If $R_M << 1$ the fluid does not appreciably affect the B field.

We thus have two parameters to consider when discussing magneto plasmas:

The magnetic Reynolds number R_M tells us whether the field is frozen in the plasma or not. The parameter β tells us whether the motion of the combined plasma-field system is dominated by the plasma (large β) or by the magnetic field (low β).

E References

Physics of Fully Ionized Gases, Lyman Spitzer, Interscience Publishers, NY. , 1956.

Introduction to Plasma Physics, Francis Chen, Plenum Press, N. Y., 1984

Fundamentals of Atomic Physics, Atam Arya, Allyn and Bacon, Inc., Boston, 1971.

Atomic and Space Physics, Alex E Green and Phillip Wyatt, Addison-Wesley Pub. Co., Inc., Reading, MA, 1965.

Introduction to Modern Physics, F. K. Richtmyer, E. H. Kennard, John N Cooper, McGraw Hill Book Co., NY, 1969.

Physics of Space Plasmas, George K Parks, Addison-Wesley, Redwood City, CA, 1991.

Plasma Diagnostics, edited by W. Lochte-Hotgreven, Wiley & Sons, NY, 1968.

F Problems

1. What is the energy per photon for a 2 giga-Hertz (GH) signal, in eV? In Joules?

2. What is the frequency of a 1 mm wavelength electromagnetic wave?

3. For an (ideal) spherical satellite, of radius 1 meter, calculate the amount of power radiated into space for a vehicle operating at $20\ ^\circ\mathrm{C}$. Remember you have to convert to degrees Kelvin.

4. At what wavelength does the radiation from the above satellite peak? If it was a re-entry vehicle at $1000\ ^\circ\mathrm{C}$, at what wavelength would the radiation peak?

5. Calculate the equilibrium temperature of a satellite in free space, at a distance from the sun of 1 astronomical unit (1 AU = 93 million miles=1.5 x 10^8 km , Solar Radius $R_\odot$ = 6.96 x 10^5 km.). Take the emissivity, ε, to be one for the sun (a good assumption), and the satellite (less so).

a) Calculate the radiated power: $P = \sigma\ T^4_{\mathrm{sun}}\ \bullet 4\pi R^2_{sun}$

b) Calculate the amount of power which reaches earth - it drops off as Gauss' law might suggest:

$$P = \sigma T^4_{\mathrm{sun}} \bullet 4\pi R^2_{sun} \bullet \frac{1}{4\pi R^2_{\mathrm{Earth\ Orbit}}}.$$

You can check that this gives the known answer: 1370 $\pm$ 4 W/m^2

c) The radiation incident on the satellite must be re-radiated:

$$P_{\mathrm{incident}} = \sigma T^4_{\mathrm{sun}} \bullet 4\pi R^2_{sun} \bullet \frac{1}{4\pi R^2_{\mathrm{Earth\ Orbit}}} \bullet \pi R^2_{\mathrm{satellite}} = \sigma T^4_{\mathrm{satellite}} \bullet 4\pi R^2_{\mathrm{satellite}}$$

noting that the radiation is incident on the projected area of the sphere, which is the circular disk cross-section. Solve the last equation for the satellite temperature.

6. For a 'Bohr' hydrogen atom, calculate the energy for the n = 3 to n = 2 transition. (Express the answer in eV). What frequency (and wavelength) does this correspond to ? Is this wavelength visible?

7. The temperature of the gas in the upper atmosphere can reach $1000\ \mathrm{K}$. Assume the region at this temperature is composed of hydrogen atoms. What is the average kinetic energy of one atom, in eV? What is the velocity of this atom (km/s)?

8. Evaluate the Maxwellian distribution, n(K), for values of K = 0.1 kT, 0.5 kT, 1.5 kT, and 10 kT. Take $N = 1 \times 10^6$, kT = 1.0 for convenience.

9. Calculate the Debye Length for a plasma with a temperature of 1,000 K, density = 1×10^5 electrons/cm^3. (upper ionosphere, altitude about 1000 km). Calculate the Debye length at geosynchronous orbit: T = 500 eV, n = 1 electron/cm^3.

10. For a sphere of radius 0.1 m, with a charge of $1.11\times10^{-10}\ \mathrm{C}$ (111 pico-coulombs), plot the electric potential as a function of distance, from r = 0.1 m, to r = 2.0 m, in vacuum. On

the same plot, plot the potential if there is a plasma present, with density 2.0×10^8 electrons / m^3, and a temperature of 1000 K. Repeat for a temperature of 1 eV.

Note that for an object of non-zero radius, the formula in the text needs to be recast into the form:

$\Phi = \frac{q}{4\pi\varepsilon_o r} \exp\left(-\frac{r - r_o}{\lambda_D} \right)$. Here, the correction factor of $\exp\left(\frac{r_o}{\lambda_D} \right)$ causes the potential at the surface of the sphere to be the same as it would have been, in the absence of a plasma.

11. Calculate the gyrofrequency (in Hz, or revolutions/second) for a proton in a magnetic field of 100 nano-Tesla (nT). (Note: a nT is also referred to as gamma, γ.)

12. Calculate the gyrofrequency (in Hz) for an electron near the earth's surface, near the equator. (You need to look up the earth's magnetic field....try chapter 4)

12. Calculate the drift velocity for a 5 eV electron, given an electric field of 1 milliVolt/meter (mV/m), which is perpendicular to a magnetic field of 200 nT. How does the drift velocity vary with energy?

13. Calculate the plasma pressure for the typical plasmasphere conditions: ion density = electron density = 2.0×10^{10} electrons / m^3, ion temperature = electron temperature = 0.5 eV. Calculate the magnetic energy density for a field of 3 micro-Tesla $(3\ \mu\mathrm{T})$. Calculate beta (β).

This page intentionally left blank